Vidyamandir Classes
Gurukul for IITJEE & Medical Preparation

Integral Calculus – 1

Level – 1	Daily Tutorial Sheet

1. **(i)** $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx=\int\left(x+\frac{1}{x}-2\right)dx=\int xdx+\int\frac{1}{x}dx-\int 2dx=\frac{x^2}{2}+\ell nx-2x+C$

(ii) $\int\frac{x^3+3x^2+4}{\sqrt{x}}dx=\int x^{5/2}dx+\int 3x^{3/2}dx+\int 4x^{-1/2}dx=\frac{x^{7/2}}{7/2}+3.\frac{x^{5/2}}{5/2}+\frac{4x^{1/2}}{1/2}+C$

(iii) $\int\frac{dx}{\sqrt{ax+b}-\sqrt{ax+c}}=\int\frac{\sqrt{ax+b}+\sqrt{ax+c}}{b-c}dx=\frac{1}{(b-c)}\left(\frac{2}{3}\frac{(ax+b)^{3/2}}{a}+\frac{2}{3}\frac{(ax+c)^{3/2}}{a}\right)+C$

(iv) $\int\frac{x^3+3x^2+2x+1}{x-1}dx=\int\left(x^2+4x+6\right)dx+\int\frac{7}{x-1}dx=\frac{x^3}{3}+\frac{4x^2}{2}+6x+7\log|x-1|+C$

(v) $\int\cos^3 xdx=\int\frac{\cos 3x+3\cos x}{4}dx=\frac{1}{4}\left[\frac{1}{3}\sin 3x+3\sin x\right]+c$

(vi) $\int(\tan x+\cot x)^2 dx=\int\left(\tan^2 x+\cot^2 x+2\right)dx=\int\left(\sec^2 x+\mathrm{cosec}^2 x\right)dx=\tan x-\cot x+C$

2. **(i)** $\int\sin x\sqrt{1-\cos 2x}\,dx=\int\sin x.\sqrt{2}(\sin x)dx=\sqrt{2}\int\sin^2 xdx=\sqrt{2}\times\frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]+C$

(ii) $\int\left(e^{a\log x}+e^{x\log a}\right)dx=\int\left(x^a+a^x\right)dx=\frac{x^{a+1}}{a+1}+\frac{a^x}{\log a}+C$

(iii) $\int\cos x.\cos 2x.\cos 3x\;dx=\int\frac{1}{2}(\cos 3x+\cos x)\cos 3xdx=\frac{1}{2}\left[\int\cos^2 3xdx+\int\cos x\cos 3xdx\right]$

$=\frac{1}{4}\left[\int(1+\cos 6x)dx+\int\cos 4xdx+\int\cos 2xdx\right]=\frac{1}{4}\left(x+\frac{\sin 6x}{6}+\frac{\sin 4x}{4}+\frac{\sin 2x}{2}\right)+C$

(iv) $\int(1+x)\sqrt{1-x}\,dx=-\int(-1-x)\sqrt{1-x}\,dx=-\int(-2+1-x)\sqrt{1-x}\,dx=\int 2\sqrt{1-x}\,dx-\int(1-x)^{3/2}dx$

$=-\frac{4}{3}(1-x)^{3/2}+\frac{2}{5}(1-x)^{5/2}+C$

3. **(i)** $\int\frac{2x-3}{\sqrt{2x+1}}dx=\int\sqrt{2x+1}-4\int(2x+1)^{-1/2}dx=\frac{(2x+1)^{3/2}}{\frac{3}{2}.2}-4\frac{(2x+1)^{1/2}}{\frac{1}{2}.2}+C$

(ii) $\int\frac{\sin x+\cos x}{\sqrt{1+\sin 2x}}dx=\int\frac{\sin x+\cos x}{\sqrt{(\sin x+\cos x)^2}}dx=\int\frac{\sin x+\cos x}{|\sin x+\cos x|}dx$

$$=\begin{cases} x+C & \forall\ x\in\left(2n\pi-\frac{\pi}{4},\,2n\pi+\frac{3\pi}{4}\right)\\ -x+C & \forall\ x\in\left(2n\pi+\frac{3\pi}{4},\,2n\pi+\frac{7\pi}{4}\right)\end{cases}$$

(iii) $\int \sin x \cos x \left(\sin 2x + \cos 2x\right) dx = \frac{1}{2}\int \sin^2 2x \, dx + \frac{1}{4}\int \sin 4x \, dx = \frac{1}{4}\left(x - \frac{\sin 4x}{4} - \frac{\cos 4x}{4}\right) + C$

(iv) $\int \frac{\sin^2 x}{\left(1+\cos x\right)^2} dx = \int \frac{4\sin^2 \frac{x}{2}\cos^2 \frac{x}{2}}{4\cos^4 \frac{x}{2}} dx = \int \tan^2 \frac{x}{2} dx = \int \left(\sec^2 \frac{x}{2} - 1\right) dx = \frac{\tan \frac{x}{2}}{1/2} - x + C$

(v) $\int \frac{1}{1-\sin x} dx = \int \frac{1+\sin x}{\cos^2 x} dx = \int \sec^2 x \, dx + \int \sec x \tan x \, dx = \tan x + \sec x + C$

(vi) $\int \frac{1}{1-\cos x} dx = \int \frac{1+\cos x}{\sin^2 x} dx = \int \operatorname{cosec}^2 x dx + \int \operatorname{cosec} x . \cot x \; dx + C = -\cot x - \operatorname{cosec} x + C$

4. **(i)** $\int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x dx}{e^{2x}+1}$; Let $e^x = t \Rightarrow e^x \, dx = dt = \int \frac{dt}{t^2+1} = \tan^{-1} t + c = \tan^{-1} e^x + C$

(ii) Let $1 - x^3 = t \Rightarrow -3x^2 \, dx = dt$

$= \int \frac{(1-t).dt}{t^2}\left(-\frac{1}{3}\right) = \int \frac{(t-1)}{3t^2} dt = \frac{1}{3} \ell n \, |t| + \frac{1}{3t} + C = \frac{1}{3} \ell n \, |1 - x^3| + \frac{1}{3\left(1-x^3\right)} + C$

(iii) $\int \frac{x^3}{\sqrt{x^8+1}} dx$ Let $x^4 = t \Rightarrow 4x^3 \, dx = dt = \frac{1}{4}\int \frac{dt}{\sqrt{1+t^2}} = \frac{1}{4} \ell n \left| x^4 + \sqrt{1+x^8} \right| + C$

(iv) $\int \frac{2x \tan^{-1} x^2}{1+x^4} dx = \int t dt = \frac{t^2}{2} + C$ $\left[\text{Put } \tan^{-1} x^2 = t \Rightarrow \frac{2x}{1+x^4} dx = dt\right]$

$= \frac{\left(\tan^{-1} x^2\right)^2}{2} + C$

(v) $\int \frac{\sin x + \cos x}{\left(\sin x - \cos x\right)^3} dx = \int \frac{dt}{t^3} = \frac{-1}{2t^2} + C$ [Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) \, dx = dt$]

$= \frac{-1}{2\left(\sin x - \cos x\right)^2} + C$

(vi) $\int \frac{x dx}{\left(ax^2+b\right)^p} = \frac{1}{2a}\int \frac{dt}{t^p} = \frac{-1}{2a\left(p-1\right)t^{p-1}} + c = \frac{1}{2a}\left(1-p\right)^{-1}\left(ax^2+b\right)^{1-p} + \text{C}$

[Put $ax^2 + b = t \Rightarrow 2ax \, dx = dt$]

5. **(i)** $\int \frac{\cos^3 x}{\sqrt{\sin x}} dx$ Let $\sin x = t^2 \Rightarrow \cos x \, dx = 2t \, dt$

$= \int \frac{\left(1-t^4\right)}{t} . 2t dt = 2t - \frac{2t^5}{5} + c = 2\sqrt{\sin x} - \frac{2}{5}\left(\sin x\right)^{5/2} + c$

(ii) $\int \cos x . \tan^3 x \, dx$ Let $\cos x = t \Rightarrow -\sin x dx = dt$

$= \int \sin x . \frac{\sin^2 x}{\cos^2 x} dx = -\int \frac{\left(1-t^2\right)}{t^2} dt = \frac{1}{t} + t + c = \sec x + \cos x + c$

(iii) $\int \tan x . \sec^6 x\, dx = \int t^5 dt = \frac{t^6}{6} + c = \frac{\sec^6 x}{6} + c$ [Let *secx* = *t* $\Rightarrow$ *secx. tanxdx* = *dt*]

(iv) $\int \tan^3 x . \sec^5 x\, dx = \int \left(t^2 - 1\right).t^4\, dt$ [Let *secx* = *t* $\Rightarrow$ *secx. tanxdx* = *dt*]

$$= \int \left(t^6 - t^4\right) dt = \frac{t^7}{7} - \frac{t^5}{5} + c = \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + c$$

(v) $\int \frac{dx}{x \cos^2 (\log x)} = \int \frac{dt}{\cos^2 t} = \int \sec^2 t\, dt$ [Let *logx* = *t* $\Rightarrow$. *dx* = *dt*]

$$= tant + c = tan(log\ x) + c$$

(vi) $\int \frac{x^3 dx}{\left(x^2 - a^2\right)^{3/2}} = \int \frac{\left(t^2 + a^2\right).tdt}{t^3}$ [Let $x^2 - a^2 = t^2$ $\Rightarrow$ $2xdx = 2tdt$]

$$= \int \left(1 + \frac{a^2}{t^2}\right) dt = t - \frac{a^2}{t} + c = \sqrt{x^2 - a^2} - \frac{a^2}{\sqrt{x^2 - a^2}} + c$$

6. $\int \frac{e^{2x}\, dx}{e^{2x} + 1} = \frac{1}{2}\, ln\, |\, e^{2x} + 1 | + c$

7. $\int \frac{dx}{\sin (x - a) \sin (x - b)} = \frac{1}{\sin (b - a)} \int \frac{\sin (\overline{x - a} - \overline{x - b})}{\sin (x - a) \sin (x - b)}$

$$= \frac{1}{\sin (b - a)} \int \frac{\sin (x - a) \cos (x - b) - \cos (x - a) \sin (x - b)}{\sin (x - a) \sin (x - b)}$$

$$= \frac{1}{\sin (b - a)} \int [\cot (x - b) - \cot (x - a)]\, dx = \frac{1}{\sin (b - a)}\ ln \frac{\sin (x - b)}{\sin (x - a)} + c$$

8. $\int \left[1 + 2 \tan^2 x + 2 \tan x \sec x\right]^{1/2} dx = \int \left[(\sec x + \tan x)^2\right]^{1/2} dx = \int \sec x\ dx + \int \tan x\ dx$

9. $\frac{1}{\sin (b - a)} \int \frac{\sin (b - a) dx}{\cos (x - a) \cos (x - b)} = \frac{1}{\sin (b - a)} \int \frac{\sin (\overline{x - a} - \overline{x - b})}{\cos (x - a) \cos (x - b)}$

$$= \frac{1}{\sin (b - a)} \int \left[\tan (x - a) - \tan (x - b)\right] dx$$

10. $\int \frac{1}{\sqrt{x + a} + \sqrt{x + b}} = \int \frac{\sqrt{x + a} - \sqrt{x + b}}{a - b}\, dx$ [Use rationalisation]

11. Rationalise to get: $I = \frac{1}{a - b} \left[\underset{\text{Put } x + a = t^2}{\int x \sqrt{(x + a)}\, dx} - \underset{\text{Put } x + b = t^2}{\int x \sqrt{x + b}\, dx}\right]$

12. $\int \sec^2 x\ \operatorname{cosec}^2 x dx = \int \frac{4}{\sin^2 2x} dx = 4 \int \operatorname{cosec}^2 2x dx = 4 \frac{(-\cot 2x)}{2} + c$

13. $\int \frac{\sin (x - a + a)}{\sin (x - a)}\, dx = \int \frac{\sin (x - a) \cos a + \cos (x - a) \sin a}{\sin (x - a)}\, dx = \cos a \int dx + \sin a \int \cot (x - a)\, dx$

14.(B) $\tan 2x = \tan [(x - \alpha) + (x + \alpha)] \quad \Rightarrow \quad \tan 2x = \dfrac{\tan(x-\alpha) + \tan(x+\alpha)}{1 - \tan(x-\alpha) \,.\, \tan(x+\alpha)}$

$\Rightarrow \quad \tan(x - \alpha) \,.\, \tan(x + \alpha) \tan 2x = \tan 2x - \tan(x - \alpha) - \tan(x + \alpha)$

$\therefore \quad \int \tan(x-\alpha) \tan(x+\alpha) \tan 2x \, dx = \int \left[\tan 2x - \tan(x-\alpha) - \tan(x+\alpha)\right] dx$

$= \dfrac{1}{2} \log|\sec 2x| - \log|\sec(x-\alpha)| - \log|\sec(x+\alpha)| + C \;=\; \log\left|\dfrac{\sqrt{\sec 2x}}{\sec(x-\alpha) \,.\, \sec(x+\alpha)}\right| + C$

15.(D) $\displaystyle\int \frac{d(\cos\theta)}{\sqrt{1-\cos^2\theta}} = \sin^{-1}(\cos\theta) + c.$ [Put $\cos\theta = x$ and integrate]